“Vibration-induced PM Noise in Oscillators and Studies of Correlation with Vibration Sensors”

National Institute of Standards & Technology (NIST), Boulder, CO, USA
Description of vibration equipment and PM noise measurement.

Primary Measurement Effects due to Motion of an Ideal Device Under Test (DUT):
- Path fluctuation
- Uniform motion
- Relativity under vibration
- Cable multipath instability and impedance mismatch

Calculation of $\Gamma = \langle y^2(f_v) \rangle^{1/2}$ per $g$

Measurements of PM noise of a low-vibe-sensitivity Qz under vibration:
- Sine-wave vibration, 20 – 2000 Hz
- Random-noise vibration, 10 – 200 Hz
- Scatter plots of phase-fluctuations vs. acceleration

Strategy for constructing ultra-low noise microwave reference oscillator with reduced acceleration sensitivity.
Vibration phase noise test bed

Vibration controller and amplifier, actuator, adaptor, and power converter
Vibration phase noise test bed

- 3-axis mounting of low-vibration-sensitivity quartz oscillator (~5x10^-11/g).
Path Fluctuations

Effects:
1. $\frac{\Delta L}{L}$ of signal path

\[
\frac{\Delta \nu_{rms}}{\nu_0} = G \cdot \frac{10}{\sqrt{2\pi \cdot 10^8}} \cdot \frac{1}{100} = G \cdot 7.3 \times 10^{-11}
\]

\[
|\Gamma| = \frac{7.3 \times 10^{-11}}{g_{rms}}, \quad @ f_{vibe} = 100Hz
\]
Uniform Motion

The detected frequency will be identical to the proper frequency of the oscillator and, to first order in the velocity, this does not depend on the velocity.
**Acceleration Effect**

**Effects:**
1. $\frac{\Delta L}{L}$ of signal path
2. 2nd order relative velocity
3. Relative acceleration

\[ \Delta v_{DUT} \frac{v_{DUT}}{v_0} = -\frac{1}{2} \frac{v^2}{c^2} \]

\[ 2.4 \times 10^{-17} / g \text{ @ } f_{vibe} = 100\text{Hz}, \]
Signal Multipath Effect from Impedance Mismatch, Dielectric Distortion, etc.

\[ V(t) = A \cos[\omega_0 t - \alpha] \]

\[ \alpha = \tan^{-1}\left( \frac{(C_0 / \beta) \cos(pt_0 / 2)}{1 - (C_0 / \beta) \sin(pt_0 / 2)} \right) \]

\[ |\Gamma| = \frac{1.46 \times 10^{-10}}{g_{rms}}, \]

in the worst case.

\( C_0 \) is an infinite series involving the coefficient of reflection \( \rho \). The quantity \( \alpha \) represents a lagging phase error that depends upon the modulation index \( \beta \). In the worst case, a reflected signal of equal strength makes \( \rho \) equal to one-half the phase lag of the reflected signal.
An oscillator that is vibrated generates carrier-frequency sidebands at $v_0 \pm f_v$ where $f_v$ is the vibration frequency. Usually $f_v$ is in the range $0 < f_v < 5 \text{ kHz}$. 
Sinusoidal vibration produces spectral lines at $\pm f_v$ from the carrier, where $f_v$ is the vibration frequency.

$$L'(f_v) = 20 \log \left( \frac{\Gamma \cdot A f_0}{2f_v} \right)$$

e.g., if $|\Gamma| = 1 \times 10^{-9}/g$ and $f_0 = 10$ MHz, then even if the oscillator is completely noise free at rest, the phase “noise” i.e., the spectral lines, due solely to a sine vibration level of 1g will be:

<table>
<thead>
<tr>
<th>Vibr. freq., $f_v$, in Hz</th>
<th>$L'(f_v)$, in dBC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-46</td>
</tr>
<tr>
<td>10</td>
<td>-66</td>
</tr>
<tr>
<td>100</td>
<td>-86</td>
</tr>
<tr>
<td>1,000</td>
<td>-106</td>
</tr>
<tr>
<td>10,000</td>
<td>-126</td>
</tr>
</tbody>
</table>
Random vibration’s contribution to phase noise is given by:

\[ L(f) = 20 \log \left( \frac{\Gamma \cdot \bar{A} f_0}{2f} \right), \quad \text{where} \quad |\bar{A}| = \left[ (2)(\text{PSD}) \right]^{\frac{1}{2}} \]

e.g., if \( |\Gamma| = 1 \times 10^{-9}/g \) and \( f_0 = 10 \text{ MHz} \), then even if the oscillator is completely noise free at rest, the phase “noise” i.e., the spectral lines, due solely to a vibration PSD = 0.1 g²/Hz will be:

<table>
<thead>
<tr>
<th>Offset freq., f, in Hz</th>
<th>( L'(f) ), in dBC/Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-53</td>
</tr>
<tr>
<td>10</td>
<td>-73</td>
</tr>
<tr>
<td>100</td>
<td>-93</td>
</tr>
<tr>
<td>1,000</td>
<td>-113</td>
</tr>
<tr>
<td>10,000</td>
<td>-133</td>
</tr>
</tbody>
</table>

Phase Deviation vs. X Axis Acceleration
10Hz to 200Hz Random Vibration
Uncompensated

\[ \Phi_{pp} = 0.02 \text{ rad} \]
Phase Deviation vs. X Axis Acceleration
Random 10 to 200Hz

\[ \Phi_{pp} = 0.0002 \text{ rad} \]
Phase Noise and Acceleration vs. Frequency

X Axis

Frequency (Hz)

Acceleration

Phase Noise

Phase Noise and Acceleration vs. Frequency

X Axis

Frequency (Hz)
X Axis

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$\phi_{pp} = 0.00035\ \text{rad}$
Phase Deviation vs. Y Axis Acceleration
10Hz to 200Hz Random Vibration
Uncompensated

$\Phi_{pp} = 0.002 \text{ rad}$
Phase Deviation vs. Y Axis Acceleration
10Hz to 200Hz Random Vibration

$\Phi_{pp} = 0.002 \text{ rad}$
Y Axis

20Hz Dwell 1g

\[ \Phi_{pp} = 0.002 \text{ rad} \]

40Hz Dwell 1g

\[ \Phi_{pp} = 0.0008 \text{ rad} \]

80Hz Dwell 1g

\[ \Phi_{pp} = 0.0005 \text{ rad} \]

100Hz Dwell 1g

\[ \Phi_{pp} = 0.003 \text{ rad} \]

150Hz Dwell 1g

\[ \Phi_{pp} = 0.0005 \text{ rad} \]

200 Hz Dwell 1g

\[ \Phi_{pp} = 0.0004 \text{ rad} \]

500Hz Dwell 1g

\[ \Phi_{pp} = 0.003 \text{ rad} \]

1kHz Dwell 1g

\[ \Phi_{pp} = 0.0003 \text{ rad} \]

2kHz Dwell 1g

\[ \Phi_{pp} = 0.0005 \text{ rad} \]
Phase Deviation vs. Z Axis Acceleration
10Hz to 200Hz Random Vibration
Uncompensated

\[ \Phi_{pp} = 0.02 \text{ rad} \]
Phase Deviation vs. Z Axis Acceleration
10Hz to 200Hz Random

$\Phi_{pp} = 0.0015$ rad
Spectral Densities, Ch. 0 and Ch. 3, at the 95% confidence interval

Log Frequency

Spectral Density

-160
-140
-120
-100
-80
-60
-40
-20
0

PSD Ch 0: Phase Detector
PSD Ch 3: Z-Axis Accelerometer
Cross-Spectral Density
Phase Noise and Acceleration vs. Frequency

Z Axis

Frequency (Hz)

Acceleration

Phase Noise

Frequency (Hz)
Z Axis
Strategy for constructing ultra-low noise microwave reference oscillator with reduced acceleration sensitivity.

While the oscillator is under vibration, an estimate of a complex-conjugate (same amplitude, opposite phase) signal will be generated from accelerometer signals and used to modulate the oscillator’s output phase in such a way as to suppress or cancel the induced sidebands. One-axis cancellation is shown for simplicity.
Conclusion

- State-of-art phase noise measurements plus vibration test facility.


The End


National Institute of Standards & Technology (NIST), Boulder, CO, USA
Conclusion

- State-of-art phase noise measurements plus vibration test facility.


Low-noise Oscillator w/ Active Vibration

Vibration-Induced Phase Noise
Nominally $5 \times 10^{-10}/g$ and $f_0 = 100$ MHz
Low-noise Oscillator w/Vibration

aPROPOS Performance Goals at 10 GHz
Nominal 1g random vibration with uniform PSD, 0.05 g²/Hz
Vibration-induced PM Noise in Oscillators and Studies of Correlation with Vibration Sensors

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The functions from accelerometers are time-domain, asymmetric signals that contain phase information in a distribution of frequencies that do not necessarily have harmonic relationships.

*Swept-sine* tests of g-sensitivity are informative, but not completely satisfactory, because harmonics of $f_v$ have phases that need to be suppressed. The g-sensitivity vs. $N \times f_v$ must include the phase of harmonics, so a superior test is *swept-triangle* at low $f_v$ to account for harmonic phase-matching. Ideally, *random noise* should be used to characterize all harmonic and anharmonic phases in the suppression.

We must resort to *adaptive transfer functions* in the feed-forward suppression (FFS).
Phase II Goal for Low-noise Oscillator w/Vibration

aPROPOS Performance Goals at 10 GHz
Nominal 1g random vibration with uniform PSD, 0.05 g^2/Hz
Phase II Goal for Low-noise Oscillator w/Vibration

aPROPOS Performance Goals at 10 GHz
Nominal 1g random vibration with uniform PSD, 0.05 g^2/Hz
Random Vibration-Induced Phase Noise

Phase noise under vibration is for $\Gamma = 1 \times 10^{-9}$ per g and $f = 10$ MHz

$\mathbf{L}(f)$ under the random vibration shown

$\mathbf{L}(f)$ without vibration

45 dB

Typical aircraft random vibration envelope
While the oscillator is under vibration, an estimate of a complex-conjugate (same amplitude, opposite phase) signal will be generated from accelerometer signals and used to modulate the oscillator’s output phase in such a way as to suppress or cancel the induced sidebands. One-axis cancellation is shown for simplicity.
Phase Deviation vs. Y Axis Acceleration
10Hz to 200Hz Random Vibration

Phase Deviation (Rad)

Y Axis Acceleration (g)

Phase Deviation (Rad)

Z Axis Acceleration (g)
Qz Osc w/ Vibration Suppression: Phase Dev. vs. Z-accel.
Qz Osc w/ Vibration Suppression: Phase Dev. vs. Y-accel.